ISI/CMI 2026

PERMUTATION & COMBINATION

Lecture 02

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SUMIT RAJPUT

📌 Olympiad & Entrance Achievements:

- PRMO Qualified – 2019
- RMO – 2020, IOQM – 2021
- ZIO (Zonal Informatics Olympiad) – 2021
- CMI Entrance (with Scholarship) – 2022

🎓 Mentored Students for Success In:

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Combinations (nCr)

A **combination** of objects is merely a **selection** (ignoring order) from a given set of objects.

In other words, a combination is just a **group or set** where the **arrangement of elements does not matter**.

$$nC_r = \frac{n \to r}{} = n(n-1)\cdots(n-r+1)(n-r)\,r! \Big/ r!(n-r)!$$

$$= \frac{n!}{r!(n-r)!}$$

Formula:

$$^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Where:

- n: total number of items
- r: number of items to choose
- !: factorial (product of all positive integers up to that number)

Properties of $\binom{n}{r}$; $0 \le r \le n$; $r, n \in \mathbb{N}_0$

(i) $\binom{n}{0} = \binom{n}{n} = 1$

(ii) $\binom{n}{r} = \binom{n}{n-r}$

(iii) If $\binom{n}{r} = \binom{n}{k}$ then $r = k$ or $n - r = k$

(iv) $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

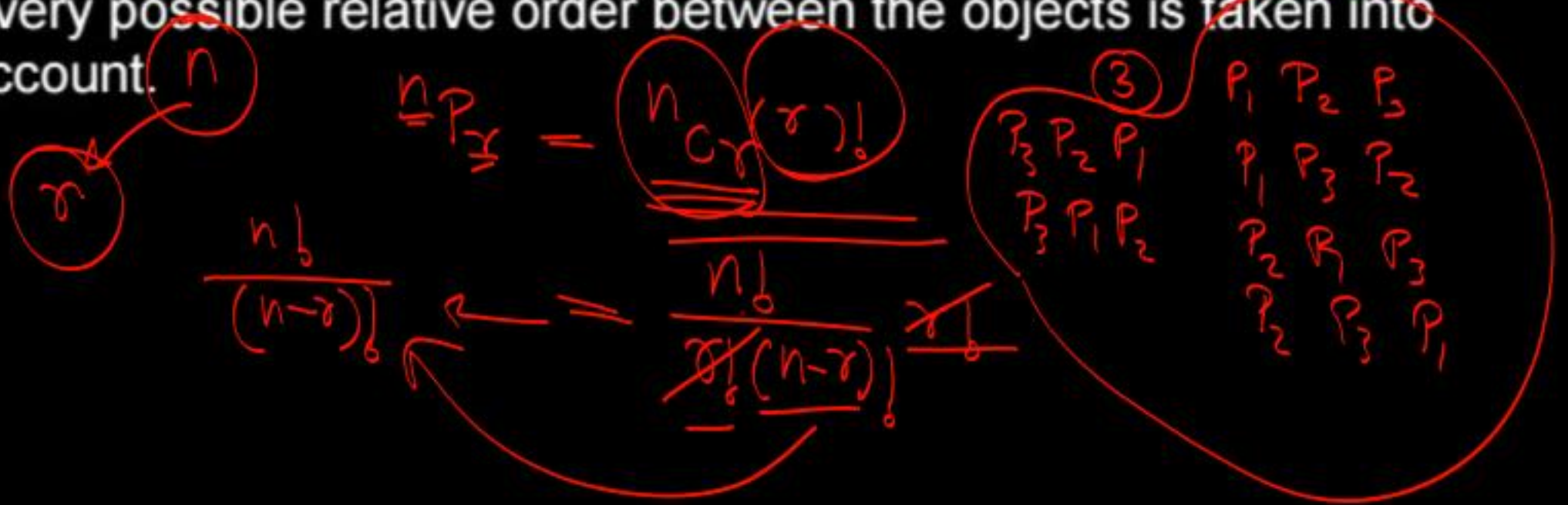
$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$${}^nC_{n-r} = \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,r!}$$

Permutation (^nP_r)

$^nC_r(r!)$

A permutation of given objects is an arrangement of the objects in a line or row, unless specified otherwise. These arrangements can be generated by changing the relative positions of objects in the row. Every possible relative order between the objects is taken into account.



Relation between permutation and combination :

$$\Rightarrow {}^{n}p_{r} = \binom{n}{r} \times r!$$

$$= \frac{n!}{r!(n-r)!} \times r!$$

$$= \frac{n!}{(n-r)!}$$

$$^{n}C_{r} = \frac{^{n}P_{r}}{r!}$$

$^{n}P_{r} = r! \, {}^{n}C_{r}$

Problem 1 : *If* ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, *find* ${}^{r}P_{2}$.

$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{\frac{56!}{(56-(r+6))!}}{\frac{54!}{(54-(r+3))!}} = \frac{56!}{54!}\,\frac{(51-r)!}{(50-r)!}$$

$$30800 = 56 \cdot 55\,(51-r)$$

$$r = 41$$

$${}^{r}P_{2} = {}^{41}P_{2} = \frac{41!}{(41-2)!} = \frac{41!}{39!} = 41 \cdot 40 = 1640$$

$$56 \cdot 55\,(51-r)$$

Solution : We have

$$\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\Rightarrow \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = \frac{30800}{1}$$

$$\Rightarrow 56.55(51-r) = 30800 \Rightarrow r = 41$$

$$\Rightarrow {}^{r}P_2 = {}^{41}P_2 = 41 \cdot 40 = 1640.$$

problem 2 : In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if

(i) there is no restriction in the selection? → $^{11}C_5$

(ii) the committee must include exactly 2 teachers? → $^{4}C_2 \cdot {}^{7}C_3$

(iii) the committee must include at least 3 teachers?

(iv) a particular teacher and a particular student cannot be both in the committee?

① $t=3$	$S=2$	$^{4}C_3 \cdot {}^{7}C_2$
② $t=4$	$S=1$	$^{4}C_4 \cdot {}^{7}C_1$

→ $^{4}C_3 {}^{7}C_2 + {}^{4}C_4 {}^{7}C_1$

t_1 is not there

+ # s_1 is not there

+ # t_1 & s_1 are not there

$$\underbrace{{}^{10}C_5} + {}^{10}C_5 + {}^{9}C_5$$

---

total − # t_1 & s_1 are included

$$\boxed{{}^{11}C_5 - {}^{9}C_3}$$

Solution. (i) The number of ways is $\binom{11}{5} = 11!/(5!6!) = 462$.

(ii) We first select 2 teachers from 4 and then $(5-2)$ students from 7. The number of ways is

$$\binom{4}{2}\binom{7}{3} = 6 \times 35 = 210.$$

(iii) There are two cases: either 3 teachers or 4 teachers are in the committee. In the former case, the number of ways is

$$\binom{4}{3}\binom{7}{2} = 4 \times 21 = 84,$$

while in the latter, the number of ways is

$$\binom{4}{4}\binom{7}{1} = 7.$$

Thus by (AP), the desired number of ways is $84 + 7 = 91$.

(iv) Let T be the particular teacher and S the particular student. We first find the number of ways to form a committee of 5 which includes both T and S. Evidently, such a committee of 5 can be formed by taking the union of $\{T, S\}$ and a subset of 3 from the remaining 9 people. Thus the number of ways to form a committee of 5 including T and S is $\binom{9}{3} = 84$.

Hence the number of ways to form a committee of 5 which does *not* include both T and S is by (CP):

$$\binom{11}{5} - \binom{9}{3} = 462 - 84 = 378, \quad \text{by (i).}$$

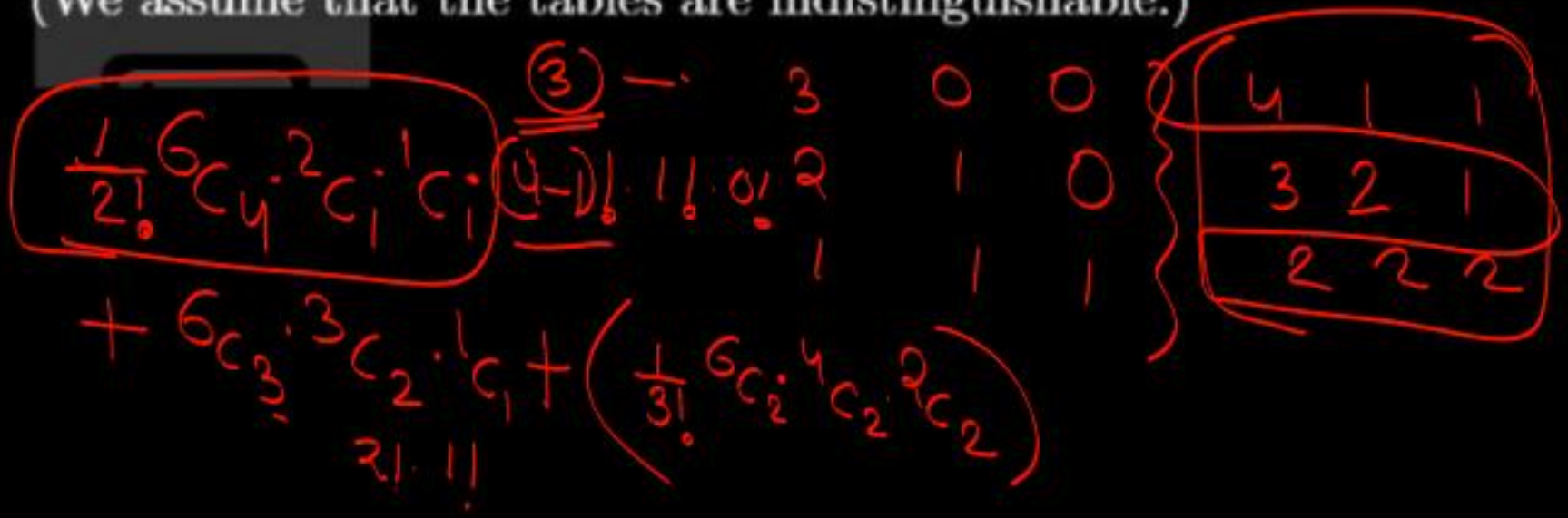
Problem 3 : If there must be at least one person in each table, in how many ways can 6 people be seated

(i) around two tables?

(ii) around three tables?

T_1 T_2 T_3

(We assume that the tables are indistinguishable.)



(i) Seating 6 people around 2 indistinguishable tables, at least 1 per table

Possible groupings:

1. $5 + 1$
2. $4 + 2$
3. $3 + 3$

Case 1 $(5 + 1)$:

$$\binom{6}{5} \cdot 4! \cdot 0! = 6 \cdot 24 \cdot 1 = 144$$

Case 2 $(4 + 2)$:

$$\binom{6}{4} \cdot 3! \cdot 1! = 15 \cdot 6 \cdot 1 = 90$$

Case 3 $(3 + 3)$:

$$\frac{1}{2}\binom{6}{3} \cdot 2! \cdot 2! = \frac{1}{2} \cdot 20 \cdot 2 \cdot 2 = 40$$

Total arrangements:

$$144 + 90 + 40 = \boxed{274}$$

(ii) Seating 6 people around 3 indistinguishable tables, at least 1 per table

Possible groupings:

1. $4 + 1 + 1$
2. $3 + 2 + 1$
3. $2 + 2 + 2$

Case 1 $(4 + 1 + 1)$:

$$\frac{1}{2}\binom{6}{4}\binom{2}{1} \cdot 3! \cdot 0! \cdot 0! = \frac{1}{2} \cdot 15 \cdot 2 \cdot 6 = 90$$

Case 2 $(3 + 2 + 1)$:

$$\binom{6}{3}\binom{3}{2} \cdot 2! \cdot 1! = 20 \cdot 3 \cdot 2 = 120$$

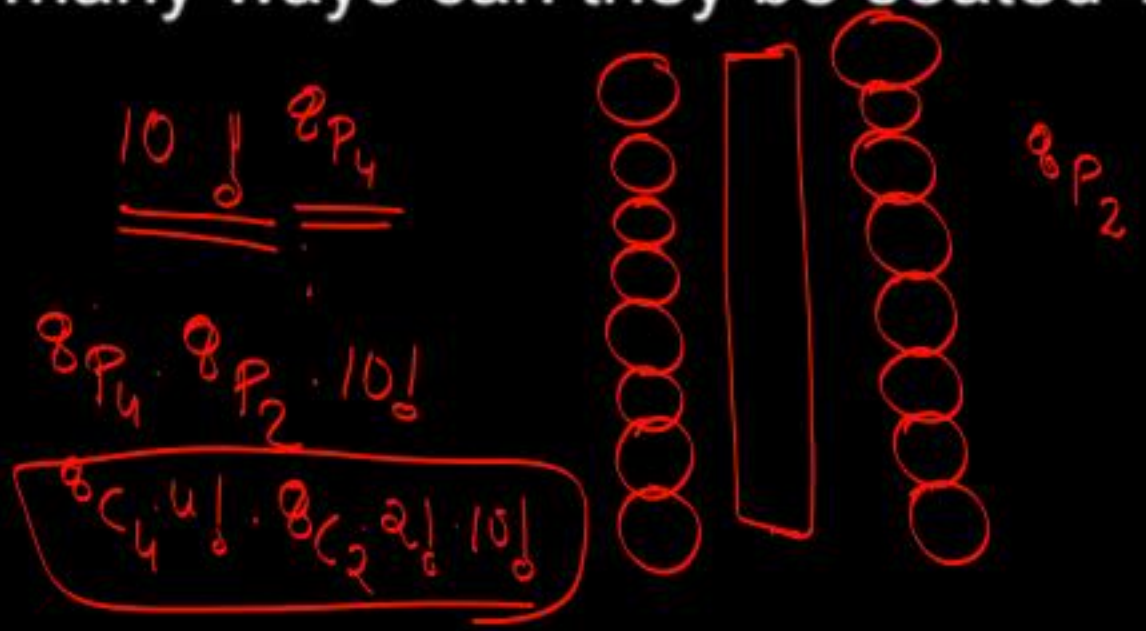
Case 3 $(2 + 2 + 2)$:

$$\frac{1}{3!}\binom{6}{2}\binom{4}{2} \cdot 1! \cdot 1! \cdot 1! = \frac{1}{6} \cdot 15 \cdot 6 = 15$$

Total arrangements:

$$90 + 120 + 15 = \boxed{225}$$

Problem 4 : A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated ?



Solution: Let $A_1, A_2, A_3, \ldots, A_{16}$ be the sixteen persons. Assume that A_1, A_2, A_3, A_4 want to sit on side 1 and A_5, A_6 want to sit on side 2.

The persons can be made to sit if we complete the following operations:

(i) Select 4 chairs from the side 1 in ${}^8C_4$ ways and allot these chairs to A_1, A_2, A_3, A_4 in 4! ways.

(ii) Select two chairs from side 2 in ${}^8C_2$ ways and allot these two chairs to A_5, A_6 in 2! ways.

(iii) Arrange the remaining 10 persons in remaining 10 chairs in 10! ways.

$\Rightarrow$ Hence the total number of ways in which the persons can be arranged

$$= ({}^8C_4 4!)({}^8C_2 2!)(10!)$$

$$= \frac{8!}{4!\,4!} 4! \times \frac{8!\,2!}{2!\,6!} 10! = \frac{8!\,8!\,10!}{4!\,6!}.$$

Problem 5 : There are 3 types of sandwiches, namely chicken (C), fish (F) and ham (H), available in a restaurant. A boy wishes to place an order of 6 sandwiches. Assuming that there is no limit in the supply of sandwiches of each type, how many such orders can the boy place?

We are given: A boy wants to order **6 sandwiches** from **3 types** — Chicken (C), Fish (F), and Ham (H). There is **no limit** on the quantity of each type.

We want to find: the number of different **ways to place an order** of 6 sandwiches.

Let:

$x =$ number of chicken sandwiches, $\quad y =$ number of fish sandwiches, $\quad z =$ number of ham sandwiches

We need to find the number of non-negative integer solutions to:

$$x + y + z = 6$$

with $x, y, z \geq 0$

Combinatorial Representation (Stars and Bars)

We can represent the 6 sandwiches as 6 identical dots:

$$\bullet\ \bullet\ \bullet\ \bullet\ \bullet\ \bullet$$

To divide these into 3 groups (C, F, H), we insert **2 dividers** (bars) between or around the dots. Each unique arrangement of the 6 dots and 2 dividers represents a different valid order.

Examples:

- ∙ ∙∙ | ∙∙∙ |∙ ⇒ 2C, 3F, 1H
- ∙ | ∙∙∙∙∙ |∙ ⇒ 0C, 5F, 1H
- ∙ ∙∙∙∙∙∙ || ⇒ 6C, 0F, 0H

Each valid order corresponds to a sequence of:

$$6 \text{ dots} + 2 \text{ dividers} = 8 \text{ total symbols}$$

Counting the Arrangements

The number of ways to arrange these 8 symbols where 2 are dividers (identical) and 6 are sandwiches (identical) is:

$$\binom{8}{2} = 28$$

Home Work :

Solve exercise 1 - 5 of Combinatorics (chapter 7) Vedantu IOQM book

Number of Questions:

- Exercise 1 : 29 questions
- Exercise 2 : 17 questions
- Exercise 3 : 13 questions
- Exercise 4 : 6 questions
- Exercise 5 : 10 questions

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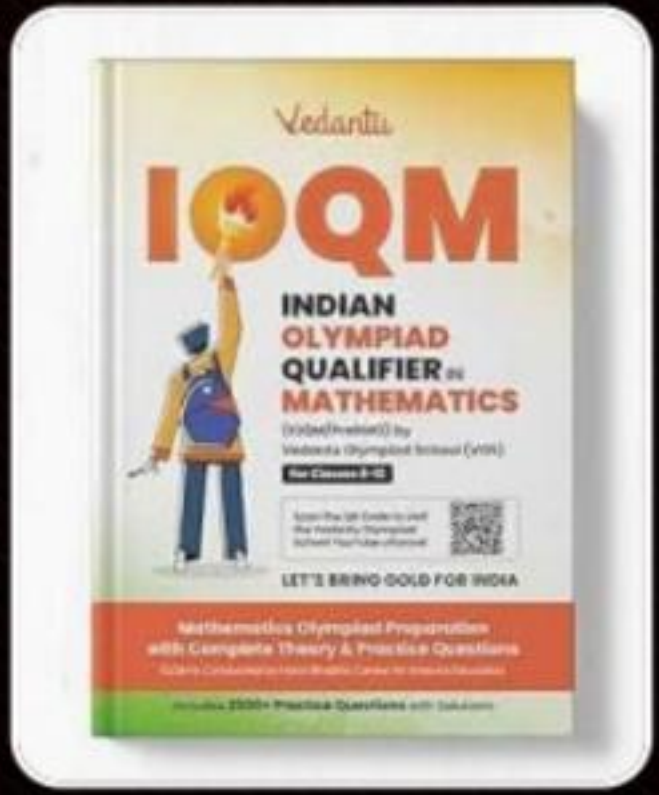
- Problem Solving Strategies-Arthur Engel
- Problem Plus in IIT Mathematics-A Das Gupta



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